Comparative Study of Warren Truss Bridge Structural Integrity

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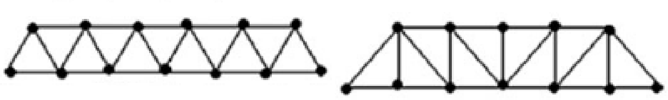
***Abstract*—In this work, the efficiency of truss in structural mechanics is investigated, with a focus on Warren-type truss for bridge design. A simulation was conducted between two models, one with and one without truss respectively, and was modelled as a linear elasticity problem by applying uniform weight until the structure material reaches its yield point. The results show that optimizing a bridge model with planar, Warren-type truss results in an improvement of almost 6 times when compared to a model without diagonal truss members. This confirms the theory that a truss is an efficient method of distributing the load on a structure. The improvements are presented using qualitative and quantitative data gathered from running a condition sweep simulation.**

***Index Terms*—Finite-Element Method, Civil Engineering, Structural Mechanics, Truss Elasticity Modeling**

I. Introduction

The weight of a structure plays an important role in the technical design of mechanical structures. Whether it may be aeroplanes, buildings, bridges, or automobiles, the produced artefact carries its own weight which is an added cost that can be minimized by practising smarter design principles. Aeroplanes and cars will consume more fuel and add to more operational costs if weight is not optimized. Buildings and bridges will require more construction material that will add extra stress to themselves to achieve greater structural integrity if the design of the artefact is not optimized for shape and topology [1]. Such optimizations can be simulated using computer-aided design (CAD) models of a given artefact and the finite element analysis (FEA) method.

Truss bridges have been designed and built for centuries in North America and around the world. Their efficient and sturdy design allows loads to be evenly distributed throughout the structure by axial forces. Two of the most common truss bridge designs are Warren and Pratt [2], as depicted in Figure 1.



(a) Warren Truss (without vertical beams), figure from [3]

(b) Pratt Truss, figure from [3]

Fig. 1: Truss Types

This specialized field of optimizing weight through build design is known as lightweight engineering [1]. The following work scopes down on the evaluation and comparison of two bridge designs using a common structural analysis method called *linear elastic analysis* (LEA) [4] and FEA to model the stress. This study aims to investigate the efficiency of truss-based bridge design through a comparison of two different bridge models, one with and one without diagonal Warren-type truss, as depicted in Figures 4 and 5. [5] [1] [6]

II. Theoretical Background

A. Analysis of Force Distribution in Planar Truss Structures

Two and three-dimensional truss structures are combined using one-dimensional members by creating triangular patterns for 2D and tetrahedral for 3D. The points of connection are referred to as joints through which the compressing and tensing forces distribute throughout the structure [3]. This results in a rigid pattern which has the nature of distributing the forces throughout itself in a uniform, axial fashion.

As trusses are components of larger structures such as buildings or bridges, the analysis of truss forces is based on the concept of *structural idealization* that determines its main components to model how forces are transmitted throughout [3]. For the truss bridge depicted in Fig. 4, the key components are two planar trusses, bracing at the top, and the flooring.

A basic analysis of planar truss is based on the simplifications that ensure forces in the truss members are axial [3].

- Loads and displacement is applied only on nodes
- Each member is straight and is arranged such that the centre of its axis lines up with the connecting node
- Nodes are connected with *pins* meaning that members can rotate

1) Method of Joints - Planar Truss: Each joint of a truss structure is subjected to concurrent forces. Equilibrium of 2D concurrent forces is expressed by following two functions, the sum of all forces in X-Y directions is equal to 0. The consequence of this is that, at most, two force unknowns can be solved for a particular joint.

$$\sum F_x = 0 \qquad \sum F_y = 0 \tag{1}$$

The approach for the method of joints is to traverse the structure from joint to joint, starting with the free-body diagram of a joint that has two unknowns, solving these unknowns and then proceeding to the next eligible joint while using the acquired force to aid the next calculation.

B. Common Construction Material for Metal Bridges

During the mid-late 19th century, steel was invented as a more stronger material than iron. The first all-steel truss bridge in USA was constructed in 1879 for the Chicago and Alton Railroad [3]. Steel has been the dominant material of choice for truss bridges until the about mid-20th century, after which cable-stayed and segmented concrete designs emerged as competitors [3]. Based on this information, the comparative study will assume that the truss-bridge models are also constructed out of steel, due to it being the dominant material of choice during the era of truss-based bridge designs. Specifically, the AISI 1020 cold-rolled steel alloy will be taken as the reference material [7].

C. Structural Stability

If the number of reactions is insufficient to resist forces applied to a structure to satisfy the equilibrium conditions, the structure is said to be initially unstable. Even if a structure is adequately supported, it may still be initially unstable if the members are not properly connected to provide sufficient internal forces to resist the applied external force [3]. Consider Fig. 2(a), it shows a rectangular structure to which force *P* is applied and is considered instable. Fig. 2(b, c) depict the same rectangular structure that is stabilized by an additional diagonal element.

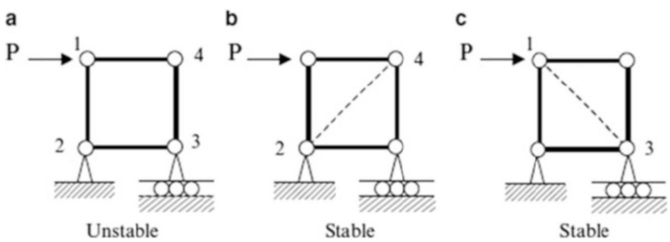


Fig. 2: Illustration of three structures with varied stability. Bottom left corner of each rectangle is fixed and bottom right corner is connected to a roller [3].

D. Maximum Distortion Energy Theory

Maximum Distortion Energy Theory is a theory of failure also known as von Mises yield criterion. It is the most popular failure theory for predicting yielding for ductile materials, and it states that a material yields once the maximum distortion energy equals the distortion energy at the yield point in a uniaxial tension test [5]. For the structural evaluation, maximum distortion energy will be used as the failure theory.

It is described mathematically by the following formula

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_{yp}^2 \qquad (2)$$

where:

$\sigma_{1,2,3}$ = three mutually perpendicular principal stresses
σ_{yp} = stress at yield point

E. Load Limit for Ductile Materials

A primary concern for designing structures is to estimate the amount of load that it can take before losing stability. This is done through identifying the load extremes and ensuring that the forces subjecting the structure satisfy the conditions for equilibrium. For truss, the instability can occur due to material failure which is described using a stress-strain curve [3]. Different stages of material stress are illustrated in Fig. 3.

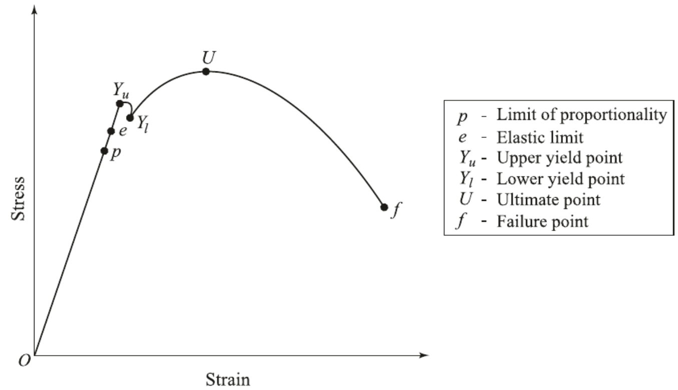


Fig. 3: Illustration of a stress-strain curve for ductile materials [5].

Fig. 3 shows that the initial stress of ductile material changes linearly as strain is applied up to the *yield point*, whose linear behaviour is also described by Young's modulus [5]. Point *'e'* is the elastic limit up to which the material recovers its original shape, for most materials points *'p'* and *'e'* are nearly identical. The stress at which the upper yield point is reached is when the material permanently deforms and is considered the point of failure [5]. In practice, the maximum allowed strain is limited to a multiple of yield strain which is called the ductility ratio and, for linear modelling, is a multiple of 5 [3].

This lays the foundation for determining the threshold of maximum allowed weight during the structural analysis in Section IV.

F. Finite Element Analysis

The finite element analysis (FEA) is a numerical technique used for solving partial differential equations by approximating a solution for given boundary conditions. Since modelling using a pen-and-paper approach oftentimes include dramatic simplification and idealization, the FEA is a strong analysis method because it allows working with design geometry with higher accuracy. Civil engineers use FEA for the analysis of beams, frames, fluids and others [8].

FEA works by reducing basic unknowns to a finite number by dividing the solution region into smaller parts called elements and expressing field variables as approximating functions for each element [8]. For solid mechanics, this can be expressed as:

$$[k]_e \delta_e = F_e \tag{3}$$

where:

$[k]_e$ = element stiffness matrix
δ_e = nodal displacement vector of the element
F_e = nodal force vector

III. Overview of the Implementation

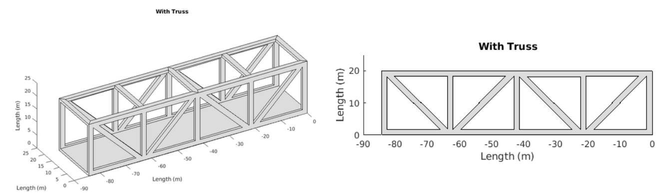
For the FEA, Matlab was used with its proprietary *Partial Differential Equation Toolbox* framework that provides useful abstractions for performing the simulation by solving partial differential equations, importing 3D models and visualizing the output [9]. Due to the near-linear behaviour of stress-strain for ductile materials up until the yield point [5], the simulation is modelled as a linear problem. Due to computational restraints, the models were imported and discretized by Matlab with a maximum element size of 1m and a minimum of 0.5m.

Two identical bridge models were imported in Matlab for processing with the only difference that one model has Warren type trussing and the other does not have any triangular truss members, see Figures 4. and 5. The evaluation is performed by analysing the maximum load that the structures can take before reaching permanent deformation (yielding). The simulation is stopped once at least one finite element of the structure has been approximated to the previously stated material stress point. To better illustrate the total force that both structures can take up before yielding, gravity is neglected for the simulation. As for the material properties, they were set as per an online database for the AISI 1020 steel alloy [7]. As the boundary conditions, both ends of the model were fixed.

Condition sweep with the *weight* parameter was done to determine the limits of the structure. The simulation started with 0N and was incremented with a step size of 980N, approximately 100kg of mass. The weight was applied uniformly to the flooring of each model.

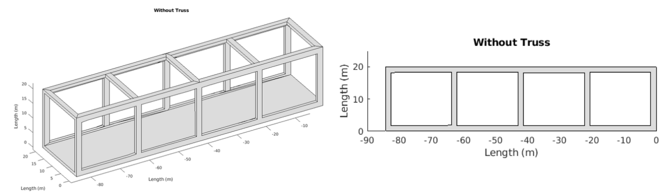
A. Bridge Models

The bridge model with trussing was obtained from an open-source resource [10]. Further modifications to remove the diagonal trussing were made using Blender 3D modelling software. Both bridge dimensions are 84.4x20x20m (LxWxH), with vertical beams of 16.4x1x1.73m; diagonal truss members at 23.2x1x2.53m



(a) Perspective view (b) Side view

Fig. 4: Bridge Model with Diagonal Trussing



(a) Perspective view (b) Side view

Fig. 5: Bridge Model without Diagonal Trussing

IV. Results and Evaluation

The simulation of the two models was run and weight values were noted once at least one element of the structure had reached the yielding point, an overview of the results is presented in Table I.

TABLE I: Overview of the weight applied to the models when yielding is reached

	With Truss	Without Truss
Yield point (3MPa)	639’940(N)	107’800(N)

Fig 6 depicts a plot of applied weight on both structures and their corresponding maximum stress. The results show that the model without truss has a substantially lower weight that it can carry. The yield point is reached when approximately 108kN of weight has been applied, or approximately 11 tons of mass.

On the contrary, the truss bridge model substantially exceeds the performance in terms of weight that can be carried until yielding of approximately 640kN. The truss model also has a more gentle growth of maximum stress per element while applying the weight as depicted in Fig. 6. This means that optimizing the bridge structure with Warren-type truss yields approximately an improvement of 6 times.

Illustration of the simulations once both the structures have yielded are depicted in Figures 7 and 8. The deformation is scaled five times for better illustration. Figures 8 and 7 show that the model with truss distributes the weight more uniformly, whereas the model without truss has more highly concentrated pressure points.

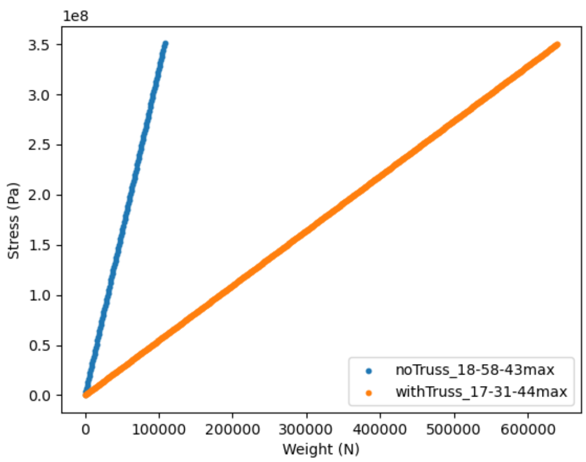
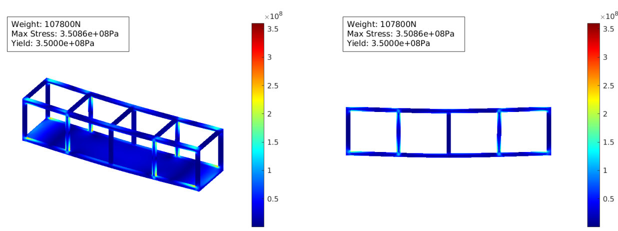
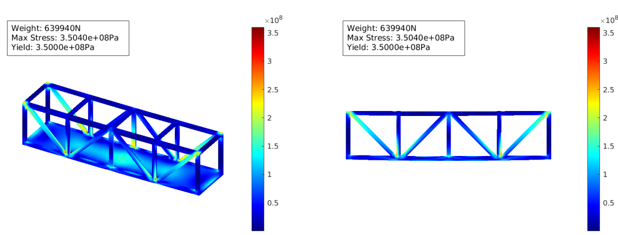


Fig. 6: Plot of the maximum stress of the structure over weight applied for both models.



(a) Perspective view (b) Side view

Fig. 7: Bridge model without diagonal trussing with approximately 108kN of force applied to it.



(a) Perspective view (b) Side view

Fig. 8: Bridge model with diagonal trussing, with approximately 640kN of force applied to it.

The greater uniform force distribution is also depicted in the histograms plots in Figures 9 and 10 respectively. It is prominent that the model with truss, has a more even number of stress occurrences per bin, as opposed to the model without truss.

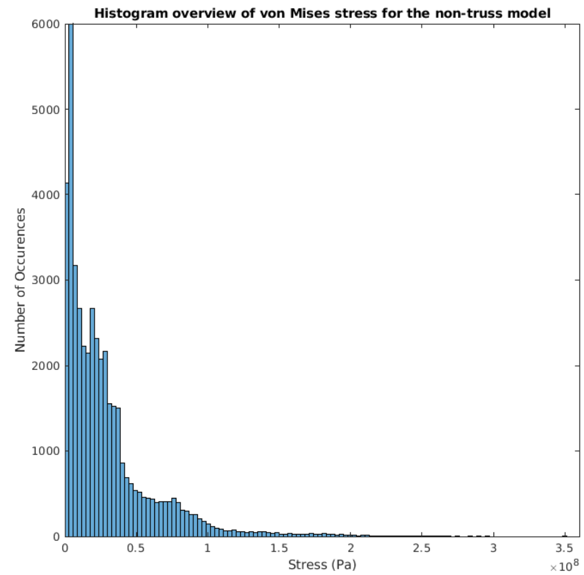


Fig. 9: Histogram overview of stress for bridge model without truss

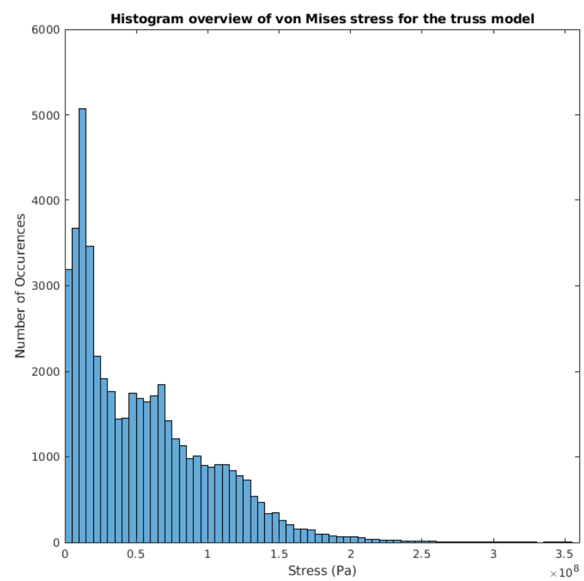


Fig. 10: Histogram overview of stress for bridge model with truss

V. Conclusions

This work investigates the efficiency of truss in structural mechanics, more specifically Warren-type truss for bridge design. The results show an improvement of almost 6 times when optimizing a structure with a planar, Warren-type truss. This confirms the theory that truss is an efficient method of distributing the load on a structure, the improvements are presented using qualitative and quantitative data.

The simulation performed in this work should not be used as an architectural recommendation, as it does not consider any safety margins on the structure as per common practise in structural engineering. As well as it also does not consider additional stresses that may occur in real-life, such as non-uniform weight distribution, gravity, or other longitudinal forces that may occur due to wind or earthquakes.

This work can further expand on modelling the problem as a non-linear elasticity problem and further investigating the structural integrity and using other material failure theories. The simulation could be continued up until the ultimate strength point of the structure as well as a complete rupture of structural members. Also, a comparative study can be carried out to compare how different truss patterns compare to a uniformly and non-uniformly distributed weight on a bridge flooring.

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